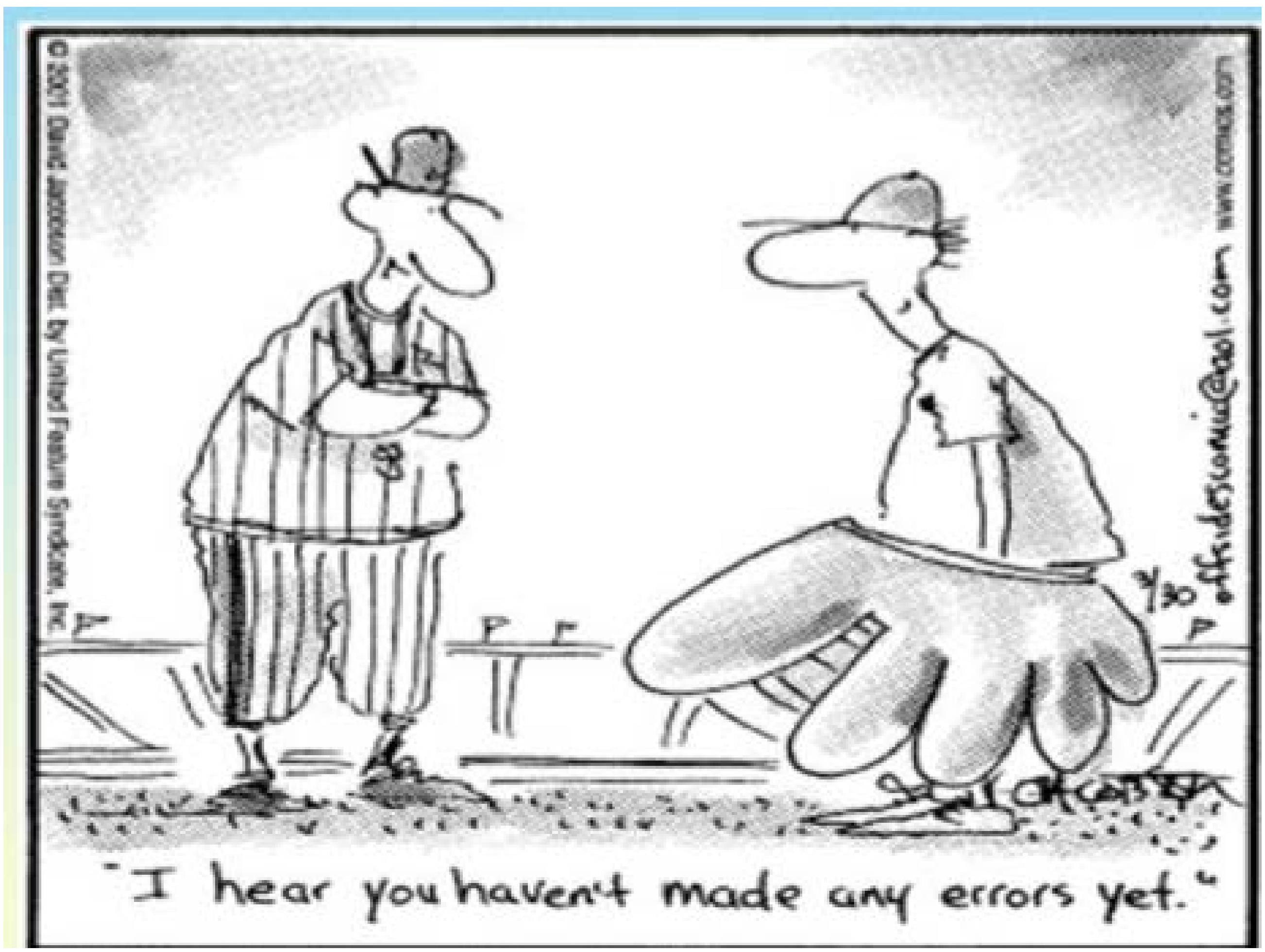
Error Analysis ~ *Project*

Name: ___________

Inquiry Question
The accuracy and precision of any recorded, scientific experiment is extremely important in science. How can you determine the overall uncertainty in your result based on the uncertainty in your measurements?

Whether you are performing a high school or university lab, or manufacturing a part for the space shuttle, errors and uncertainty continue to accumulate with more and more measurements. You need to be able to present your final results with a "tolerance" or error range. Better procedures and better measuring equipment can yield lower tolerances.

How can we calculate the final error in an equation when more than one variable contains uncertainty?



Instructions
Using a pencil, answer the following questions. The lab is marked based on clarity of responses, completeness, neatness, and accuracy. Do your best! Please ensure that any data measured (or recorded) includes the appropriate number of significant digits (only one uncertain digit).
This activity is divided into three sections: • **Core** – this first section explores only the basic “core” ideas involved in understanding. Students must demonstrate a sound understand with all of their answers in this section BEFORE attempting the next section. • **Mastery** – Your instructor will NOT review this section if the Core section above shows any misconceptions. In this section students will make predictions and apply the concepts and ideas learned above. For complete mastery it is expected that data collection and scientific procedures will be as accurate as possible. All work shown should be clear with any units included. Answers should be rounded off to the correct number of significant figures based on the data collected. • **Ace** – Once again, your instructor will only look at this section provided he/she is confident that the above Mastery criteria has been met. In this section students will demonstrate a deeper understanding of the concepts through error analysis, experimental design etc. Physics concepts from other units already covered will often be required here.
This Project will be graded according to this Marking Rubric (link).

The purpose of this activity is to introduce yourself to the mathematical rules surrounding error analysis. By the end of this activity you will be familiar with absolute error and relative error. You will be able to determine the range of error in your final calculation by simply determining the minimum and maximum possible values based on uncertainties in measurements (given). You will then use this range to determine the mathematical rule that will be used from this point on to determine overall uncertainties in your labs and calculations.

Part 1: Core

An Introduction to Error Analysis:

Each section will present the general rule that will guide you through the questions following. Work in pencil and be sure to check your answers as you go.

Absolute versus relative error:

Absolute error is the actual value of the error in physical units. For example, let's say you managed to measure the length of your cat, *L*, to be *42cm* with a precision *4cm*. This means that your cat's length might be as small as *(42-4)cm = 38cm*, and as large as *(42+4)cm = 46cm*

The convention for reporting your result with an absolute error would be

$$L = 42cm \pm 4cm$$

Where ±4cm represents the absolute error.

Notice that the measurement and the absolute error will always have the same number of decimal places. In other words, we can't write L = 41.94cm ±4cm since the error shows less precision than the measurement. We must round to the same decimal place as the error so that we get L = 42cm ±4cm. If our absolute error was only was ±0.2cm we would round to one decimal place as follows; 41.9cm ±0.2cm

Absolute error will always have the same units as the actual measurement. In general, the absolute error will always be rounded to include ***only one significant figure*** or the ***same number of decimal places as the answer*** expressed with the correct number of significant figures)

Relative error is simply the absolute error writen as a ***percentage*** of the measured value. In other words:

$$\text{Relative error} = (\text{absolute error})/(\text{measured value}) \times 100\%$$

For the cat above we could determine the relative error as follows:

$$4cm/42cm \times 100\% = 9.5\% = 10\%$$

We would write this (in terms of relative error) as

$$L = 42cm \pm 10\%$$

Questions:

1. What precise meaning do you attach to the statement "r = (24.0 ± 0.3) mm", where r is the radius of a tube?

2. How do you write T = 1.25578 s ± 0.1247 s keeping one significant digit in the error?

3. What is the relative error for v = 12.25 m/s ± 0.25 m/s ?

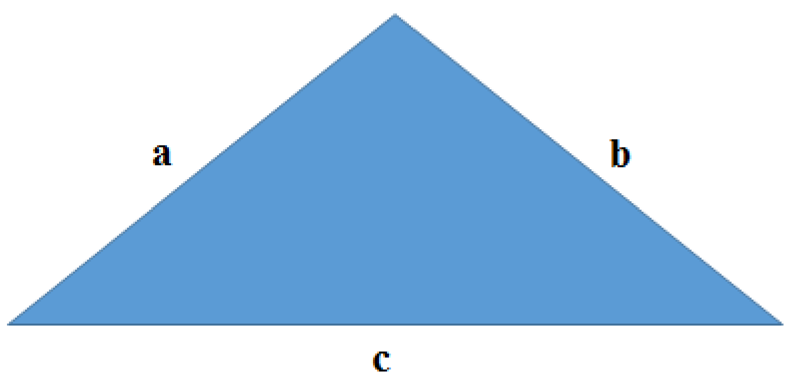
4. What is the absolute error if the central value is 121 s and the relative error is 5%?

5. The radius of this circular dartboard is measured as 9 inches, rounded to the *nearest inch*. The actual radius is 8.6 inches. What, to the *nearest percent*, is the percent of error in the measurement of the radius?

6. A measurement is taken to be 20.40 cm and the absolute error is ±0.05 cm. Find the percent of error

Errors Involving Sums and Differences:

Imagine you measure around the perimeter of the triangle below and obtain the following measurements:



a = 54 ± 5 cm

b = 52 ± 3 cm

c = 78 ± 6 cm

a. What is the largest possible perimeter based on your measurements? Show work clearly. (ans. 198 cm)

b. What is the smallest possible perimeter based on your measurements? Show work clearly. (ans. 170 cm)

c. What is the best estimate for the perimeter? Show work. (ans. 184 cm)

d. Based on our answers above we would say that our perimeter is 184 cm ±_________ cm?

e. How does the absolute error in the perimeter above relate to the absolute errors of the individual side measurements?

Rule for addition and subtraction: ***The absolute error of the result is the sum of the absolute errors of the original quantities.*** *Remember, even if you subtract two quantities you still* ***add*** *their absolute errors.*

Questions:

1. If the tolerance of a dimension on a machine part is listed as 2.54 cm ± 0.03 cm, which dimension does not meet specified tolerance? Briefly explain.
   a. 2.54 cm
   b. 2.56 cm
   c. 2.58 cm
   d. 2.51 cm

2. A computer monitor is rectangular in shape. To the *nearest inch,* the length of the monitor is 15 inches and its width to the *nearest inch* is 13 inches.

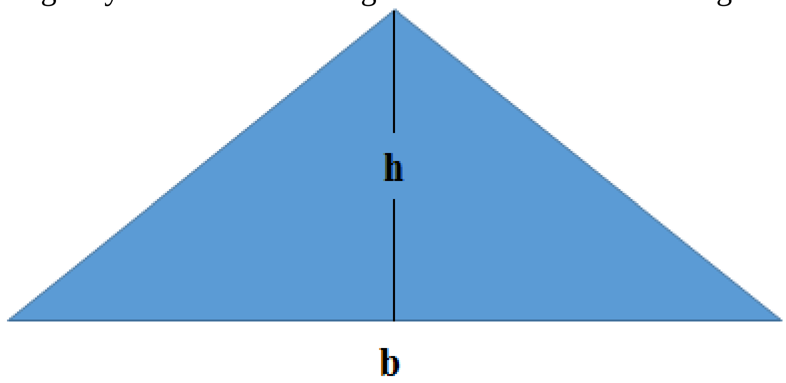
   What would the best value of the perimeter be (include the absolute error or tolerance)? What is the least possible value of the perimeter of the computer monitor to the *nearest ten*?

3. If measurement A = (12.6 ± 0.4) m/s, measurement B = (4.8 ±0.3) m/s, and measurement C = (10.35 ±0.08) m/s, what is

   a. A + B?
   b. B – A?
   c. A + B - C?

Errors Involving Products and Quotients:

Imagine you measure the height and the base of the triangle below as shown:



h = ***45 ± 3 cm***

b = ***78 ± 6 cm***

Show all work:

a. Determine the relative error for each of the measurements (maintain a few decimal places for now)

h = ___________________________

b= ___________________________

b. What is the largest possible area based on your measurements? Show work clearly. (ans. 2016 cm^2)

c. What is the smallest possible area based on your measurements? Show work clearly. (ans. 1512 cm^2)

d. What is the best estimate for the area? Show work. (ans. 1755 cm^2)

e. Based on our answers above we would say that our area is 1755 cm^2 ±__________ cm^2? (hint: you may average out your two tolerances based on the smallest and largest values)

f. Determine the relative error in your area above. (ans. 14.4%)

g. How does the relative error in the area above relate to the relative errors of the individual side measurements?

Rule for ***multiplication*** *and* ***division****:* ***The relative error of the result is the sum of the relative errors of the original quantities.***

Questions:

1. Alec measured the width and height of a rectangle, but was only able to measure them to the nearest centimeter (ie. ± 0.5cm using our rules for measuring devices). He recorded the width as 8 cm and the height as 5 cm.
   a. Determine the area A (in cm^2) of the rectangle including the total relative error?

   A = ________________ cm^2 ±_________ %

   (ans: 40 cm^2 ± 16%)

   b. Determine the area A (in cm^2) of the rectangle including the total absolute error?

   A = ________________ cm^2 ±_________ cm^2

   (ans: 40 cm^2 ± 7 cm^2)

2. Faith measured the length, width and height of a cuboid, each to the nearest cm. She recorded the length as 9 cm, the width as 2 cm and the height as 3 cm. What is the volume V (in cm^3) of the cuboid? Show all work and write your answer with both relative and absolute errors.

   (ans: 54 cm^3 ± 47% or ± 26 cm^3)

3. A rectangular block has mass (1.5 ± 0.1) kg, and dimensions (80 ± 2) mm, (50 ± 1) mm and (30 ±1) mm. Assuming that all the errors are independent, calculate the uncertainty in
   a. the volume of the block

V = _____________ mm^3 ±__________ mm^3

(ans: $1.2x10^5$ mm^3 ± 8% or ± $0.1x10^5$ mm^3. Note: we always round the absolute error to the same decimal place as the answer with the correct sig figs)

   b. the density of the block

ρ = ___________________ kg/mm^3 ±___________ kg/mm^3

(ans: $1.3x10^{-5}$ mm^3 ± 14% or ± $0.2x10^{-5}$ cm^3

Part 2: Mastery

Errors Involving Multiplication with Constants:

Bill measures the radius of a circle to be 15.0 mm ±0.4 mm. He needs to determine the circumference of this circle along with its overall uncertainty.

Using c = 2πr determine: *(show all work and keep a few decimal places for this exercise)*

a. The maximum possible circumference. *(ans: 96.76mm)*

b. The minimum possible circumference. *(ans: 91.73mm)*

c. The "best" circumference. *(ans: 94.24mm)*

d. Write your "best" circumference including the uncertainty based on the minimum and maximum values above. *(ans: 94.24mm ± 2.51mm)*

e. The constants in our formula for circumference are 2 x π. Our original absolute error in the radius was ± 0.4mm. How could we get our overall absolute error of ±2.51mm using only 2, π, and ± 0.4mm?

*Rule for **multiplying or dividing by a constant(s)**. When multiplying by a constant(s), the **absolute error is multiplied by the same constant(s).***

Or…

f. Determine the relative error in the radius. *(ans: 2.67%)*

g. Determine the relative error in your "best" circumference from part (d) above. *(ans: 2.67%)*

Rule for ***multiplying or dividing by a constant(s)****. The* ***relative error overall is the same*** *as it was for all the measurements involved in the calculations.*

Questions:

1. The top of a cylinder has a radius of r = (0.050 ± 0.005)cm. The height of the cylinder is h = (0.17 ± 0.01)cm. Determine

a. The circumference of the top of the cylinder. Provide answers with both relative and absolute errors. *(ans: 0.31cm ±10% or (0.31 ±.03)cm)*

b. The surface area of the side of the cylinder (imagine a hollow tube) using the formula below. Provide answers with both relative and absolute errors. *(ans: 0.0534*cm^2 *±16% or (0.0534 ± 0.008)*cm^2*)*

$$SA = 2\pi rh$$

Errors Involving Raising a Quantity to Some Power (eg x^2, or $x^{3/2}$):

Bill is measuring the period of motion for a mass on a spring. He measures the mass to be *m = 0.150kg ± 0.002kg*. Eventually, to determine the overall period of the spring mass oscillator, he must determine $\sqrt{m}$

a. Maximum possible value for $\sqrt{m}$? *(ans: 0.390* $\sqrt{kg}$*)*

b. Minimum possible value for $\sqrt{m}$? *(ans: 0.385* $\sqrt{kg}$*)*

c. The "best" value for $\sqrt{m}$ (include the uncertainty with a few decimals for now)? *(ans: 0.387* $\sqrt{kg}$ *± 0.0026* $\sqrt{kg}$ *)*

d. Determine the relative error in our original mass measurement. *(ans. 0.150kg ± 1.3%)*

e. Determine the relative error in the answer from part (c). *(ans: 0.387* $\sqrt{kg}$ *± 0.67%)*

f. What is the power (exponent) involved when we find $\sqrt{m}$?

Rule for raising a number to a power of n: When a number is raised to a power of n, the relative error is the multiplied by n.

g. Using the relative error you determined in part (d) and the power determined in part (f), calculate the overall relative error in $\sqrt{m}$ by utilizing the above rule. *(ans: 0.67%)*

Questions:

1. One side of a cube is measured to be $a = 34.6\ cm \pm 0.2\ cm$. What is the overall volume using $V = a^3$? Write your answer with both relative and absolute uncertainties. *(ans: ±1.7% error or ±700* cm^3*)*

V = ________________ cm^3 ± ____________ %

V = _______________ cm^3 ± ___________ cm^3

2. The area of a square is $A = 115\ m^2 \pm 7m^2$. What is the length of each side, x? Write your answer with both relative and absolute uncertainties. *(ans: ±3% error or ±0.3 m)*

x = ______________ *m* ± ____________ %

x = _______________ *m* ± ___________ *m*

3. If $r = 0.057\ m \pm 0.002m$, then what is the overall value of $\sqrt[3]{r^2}$ including uncertainties? *(ans: ±2.34% error or ±0.003 m)*

r = ______________ *m* ± ____________ %

r = _______________ *m* ± ___________ *m*

Summary of Rules:

Operation	**Type of Resulting Error**	**Found by**
Addition or Subtraction	Absolute	Add absolute errors in quantities summed or subtracted.
Multiplication or Division	Relative	Add relative errors in quantities multiplied or divided
Raising to nth power	Relative	n x relative error in original number.

Part 3: Ace

Combinations:

You will rarely be confronted with an equation that simply follows one of the rules above. More often than not you will be required to determine the overall error for an equation that utilizes a combination of the rules above.

Example:

Calculate the value of the quantity X and its maximum error ±x from the measured values of a, b and c in each of the following:

X =6a + 4b	*a =40 ± 2*	*b =20 ± 2*

For the above equation we see multiplication combined with addition.

Step 1: Get the value of X (without errors): We simply substitute in for *a* and *b*

X = 6(40) + 4(20) = 320

Step 2: Determine the relative error for each value:

a: 2/40 x 100% = 5%
b: 2/20 x 100% = 10%

Step 3: Maintaining the same "order of operation" we deal with the multiplication first (relative error). Let's do each term separately.

6a = 6(40) = 240 ± 5% convert to absolute error = 240 ± **12**
4b = 4(20) = 80 ± 10% convert to absolute error = 80 ± **8**

Step 4: Now we can deal with the addition of the first term (6a) to the second term (4b). Since this is adding/subtracting we will switch to absolute error.

(240 ± **12**) + (80 ± **8**) = 320 ± **20** (where we simply add the absolute errors)

Therefore: 6a + 4b = 320 ± 20 = 320 ± 6%

Questions: Try the questions below: Show all work

1. $X = a - 2b$	$a = 50.0 \pm 1.0$	$b = 24.0 \pm 0.5$		*Ans: 2 ± 2*
2. $X = a^3$	$a = 10.0 \pm 0.3$	$b = 0$		*Ans: 1000 ± 90*
3. $X = \frac{a}{\sqrt{b}}$	$a = 100.0 \pm 4.0$	$b = 50.0 \pm 2.0$		*Ans: 14.1 ± 0.8*
4. $X = \frac{ab^2}{c}$	$a = 0.200 \pm 0.004$	$b = 0.100 \pm 0.003$	$c = 0.050 \pm 0.002$	*Ans: 0.040 ± 0.005*

5. Suppose $F = \frac{P\pi a^4}{8lw}$:

   If $P = (101 \pm 4) N/m^2$, $a = (0.068 \pm 0.002) m$, $l = (0.040 \pm 0.001) m$, and $w = (0.060 \pm 0.001) m$ determine F and its uncertainty limits.
   (*ans: 0.35N ± 0.07N)*